About using beliefs induced by probabilities

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Abstract—**In this paper, we present a continuous approach of the theory of belief functions. We assume that the knowledge of the sources of information is represented by distributions of probability. Then we deal with the way to transform probabilities into belief functions. According the nature of the sources (objective, subjective, reliable), we apply the least commitment principle or the principle of maximum of necessity to generate a belief function from a probability. It is a way to take into account our trustworthy in a source of information. To illustrate our work, we apply the presented approach to a problem of model based classification using imprecise sensors' data and we compare the results to those obtained with a classical approach of bayesian classification. The advantage of using the theory of belief functions is that we can model the lack of *a priori* informations.**

Keywords: Continuous belief functions, classification, sensors, models, subjective/objective/reliable source.

I. Introduction

Originally, the belief functions have been used to model a set of probability distributions [6] and random sets [7]. In his famous book "*A mathematical theory of evidence*" [24], G. Shafer presents an interpretation of belief function where he assignes weights on sets to represent the opinion of an agent. From that time, the theory of belief function has been used in various fields like sensors' data fusion [1], clustering [8], ... The purpose of these works were to merge sources of informations and to take decisions within the frame of the theory of belief functions using continuous description for information. Unfortunately, they have used discret frames of discernement to represent informations on a continuous framework. In this context, it could be a good solution to use a continuous formalism to describe belief functions. In [25], a mathematical framework is suggested to describe in a general way a belief function. In particular cases (we assign weight only on intervals) [28], [32] (or linear belief function) [18], some approaches have been proposed. Some applications of these works to model based classification [3], [23], [29] and to state estimation [21] exist. Our aim is to make fusion of information coming from sensors and expert opinions. Usually, the information is modelized with distributions of probability. Hence, the stake here is to be able to associate a belief function to a probability.

To deal with such a problem, we present in section II the approach of belief functions exposed in [10]. Using this framework, we will take into account the state of the source of information to generate a belief function induced by a probability. In theory of belief functions, it is usual to apply the principle of least commitment to chose a belief function among a set of belief functions. We can use this principle when we consider the set of belief functions whose the pignistic transformation is equal to the one given by a source of information [28]. In fact, that assumes the source of information is subjective (*cf.* section III-B). Therefore we transfer the information from singletons to sets as big as possible. If we assume that the source of information is objective, we can apply the principle of maximum of necessity. It comes from the theory of possibilities [19]. In this case we decide to modelize the probability transmitted by the source with the most informative consonant belief function as possible (*cf.* section III-C). Hence we put the most information as possible on the smallest confidence sets as possible. To illustrate the interest of these several approaches to build belief functions by following these principles, we apply them to a model based classification problem using imprecise sensors' data (*cf.* section IV). The models given by experts are supposed to be subjective because we make the assumption the human expert has made a bet on the behaviour of the phenomenoms he observes. The measures transmitted by sensors are supposed to be objective because there is no subjective point of view which influences the results it gives. We decide to apply a modified version of the general bayesian theorem in decision making with *a posteriori* information. As we have no *a priori* knowledge about the occurency of event, we will compare this approach with classical probabilistic bayesian approach in order to see the efficiency of this formalism to deal with the lack of information.

II. Continous belief functions

A. Allocation of probability

In "*Allocation of probability*" [25], G. Shafer links the theory of belief functions to the "Theory of capacities" of G. Choquet [5]. He defines the belief function bel as a function on a multiplicative subclass $\mathcal{E}$ of $\mathcal{P}(\Omega)$ ($\varnothing$ and Ω are in $\mathcal{E}$) such as $bel(\varnothing) = 0$, $bel(\Omega) = 1$, bel is monotone and of order ∞. In the same way, he defines the plausibility function pl such as $pl(\complement A) = 1 - bel(A)$ and $\complement A$ is the complementary of A. That is a function on an additive subclass $\mathcal{E}^\star$ of $\mathcal{P}(\Omega)$ ($\varnothing$ and Ω are in $\mathcal{E}^\star$) such as $pl(\varnothing) = 0$, $pl(\Omega) = 1$, pl is alterning and of order ∞. He explains this kind of belief functions can be extended to $\mathcal{P}(\Omega)$ and defines two regularity conditions for these functions, the continuity and the condensability. Working on this framework, he proved that if bel is a belief function,

there exists an allocation of probability $\rho : \mathcal{E} \to \mathcal{M}$ (in fact an homomorphism) and μ a measure associated with the probability algebra $\mathcal{M}$ such as $bel = \mu \circ \rho$. That is an important result. Indeed, it allows us to use the theory of measure to compute belief functions. In this case, the real problem is to find an efficient way to describe all the elements of the multiplicative subclass $\mathcal{E}$.

B. Index function and credal measure

In [28], Ph. Smets works on continuous belief functions on real numbers. He proposes to put basic belief assignement only on the intervals of $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$. With this assumption, he had an efficient way to describe all the focal elements and made a link between belief functions on $\overline{\mathbb{R}}$ and probability density functions on $\mathbb{R}^2$ using the concept of basic belief densities. However, this framework is quite restrictive and in somes applications, we need to express beliefs on more complex sets. In [10], the authors explicit an index function to scan $\mathcal{F}$, the set of all focal elements of a belief function on $\mathcal{P}(\Omega)$, using a set I, the index space:

$$f^I : \begin{array}{rcl} I & \longrightarrow & \mathcal{F} \\ y & \longmapsto & f^I(y) \end{array} \tag{1}$$

Credal measure μ^Ω is a positive measure such as $\int_I d\mu^\Omega(y) \leqslant 1$. Then we consider that a credal space is defined by the brace $\left(f^I, \mu^\Omega\right)$. In order to compute belief functions, we need to define for all A in $\mathcal{P}(\Omega)$ the measurable sets in $\mathcal{P}(I)$:

$$F_{\subseteq A} = \{y \in I | f^I(y) \subseteq A\} \tag{2}$$

$$F_{\cap A} = \{y \in I | \left(f^I(y) \cap A\right) \neq \varnothing\} \tag{3}$$

$$F_{\supseteq A} = \{y \in I | A \subseteq f^I(y)\} \tag{4}$$

Hence we have:

- The belief function:

$$bel^\Omega(A) = \int_{F_{\subseteq A}} d\mu^\Omega(y) \tag{5}$$

- The plausibility function:

$$pl^\Omega(A) = \int_{F_{\cap A}} d\mu^\Omega(y) \tag{6}$$

- The communality function:

$$q^\Omega(A) = \int_{F_{\supseteq A}} d\mu^\Omega(y) \tag{7}$$

It can be used to model a belief on Borel algebra $\mathcal{B}\left(\overline{\mathbb{R}}^n\right)$. In this case, $F_{\supseteq A}$, $F_{\subseteq A}$, $F_{\cap A}$ must sets of $\mathcal{B}(I)$ for all A in $\mathcal{B}\left(\overline{\mathbb{R}}^n\right)$. In this case, the dimension of the index space, like in Ph. Smets works, is not linked to the dimension of the frame of discernment. We can consider the following function:

$$\tilde{f}^I : \begin{array}{rcl} \mathcal{F} & \longrightarrow & \mathcal{P}(I) \\ A & \longmapsto & F_{\subseteq A} \end{array} \tag{8}$$

This function helps us to define a homomorphism on a multiplicative subclass induced by $\mathcal{F}$ to a multiplicative subclass induced by I. Hence, there is a link between the approaches of allocation of probability and index function to describe focal elements. In this framework, we define some basic tools. Let μ_1^Ω and μ_2^Ω be two credal measures. Let combine them thanks to the conjonctive rule of combination [31]. We can build the credal measure $\mu_{1 \bigcirc 2}^\Omega$ [10] such as:

$$q_{1 \bigcirc 2}^\Omega(A) = q_1^\Omega(A) \cdot q_2^\Omega(A) \tag{9}$$

There are other ways to combine informations within the theory of belief functions. One of them is the cautious rule [17] used to combine correlated sources of information. Let f^{I_1} and f^{I_2} be two index functions linked to the credal measures μ_1^Ω et μ_2^Ω. Let φ be a change of variables such as $\varphi(y_1) = y_2$ implies $f^{I_1}(y_1) = f^{I_2}(y_2)$. The braces $\left(f^{I_1}, \mu_1^\Omega\right)$ and $\left(f^{I_2}, \mu_2^\Omega\right)$ represent the same belief if [10]:

$$d\mu_1^\Omega(y_1) = \left|det\left(\varphi'(y_1)\right)\right| d\mu_2^\Omega(\varphi(y_1)) \tag{10}$$

Within this framework, we will study a particular type of belief functions, the consonant ones.

C. Consonant belief functions

A belief function whose the focal elements are nested is a consonant belief function. This allows us to create a total ordering on $\mathcal{F}$ linked to the $\subseteq$ relation. Hence, we can define an index function f from I, a subset of $\mathbb{R}^+$, to $\mathcal{F}$ such as $(y \geqslant x) \implies (f(y) \subseteq f(x))$ [28]. The α-cuts of g, a continuous function from $\overline{\mathbb{R}}^n$ to $\mathbb{R}^+$ is the set such as:

$$f_{cs}^I(\alpha) = \{x \in \mathbb{R}^n | g(x) \geqslant \alpha\} \tag{11}$$

We can define an index function:

$$f_{cs}^I : \begin{array}{rcl} I = [0, \alpha_{\max}] & \longrightarrow & \{f_{cs}^I(\alpha) | \alpha \in I\} \\ \alpha & \longmapsto & f_{cs}^I(\alpha) \end{array} \tag{12}$$

We have the property that $F_{\subseteq A}^{cs}$ is an element of Borel algebra. Indeed:

$$\begin{aligned} F_{\subseteq A}^{cs} \neq \varnothing &\Rightarrow \exists \alpha_{\inf} = \inf\{\alpha \in I | f_{cs}^I(\alpha) \cap A \neq \varnothing\} \\ &\Rightarrow F_{\subseteq A}^{cs} = \left]\alpha_{\inf}, \alpha_{\max}\right] \end{aligned} \tag{13}$$

Using a similar argument, we can prove that $F_{\supseteq A}^{cs}$ and $F_{\cap A}^{cs}$ are elements of Borel algebra. Hence, we can define consonant belief functions whose the focal elements are given by the α-cuts of a probability density function. That will be useful to transform probabilities into belief functions. The problem of modeling a belief with a family of nested focal elements is that the combination of two consonant belief functions does not induce a consonant belief function. Solutions to this problem have been proposed in [9], [10].

III. FROM PROBABILITIES TO BELIEF FUNCTIONS

A. Pignistic transformation

In most of the cases, imperfect knowledge is modeled with probability. Unfortunately, this framework is not suitable to represent phenomenons such as ignorance or uncertainty [27].

Therefore we want to associate a belief function to a probability. There are operations to derivate a probability from a belief function. One of them is the pignistic transformation. Ph. Smets [30] has given a justication of this transformation in the transferable belief model. He proposes to use it in order to take decision on singletons. We propose to describe this transformation with the equality:

$$BetP(A) = \int_{F \cap A} \frac{\nu(A,y)}{\nu(f^I(y),y)} d\mu^\Omega(y) \qquad (14)$$

If we work on a discrete frame of discernement, $\nu(A,y)$ gives the cardinality of $A \cap f^I(y)$. We define on real numbers:

$$\nu(A,y) = \lambda\left(A \cap f^I(y)\right) + \omega(A,y) \cdot \delta\left(\lambda\left(f^I(y)\right)\right) \quad (15)$$

where λ is the Lebesgue measure, δ is the Dirac measure and $\omega(A,y)$ is a number in $[0,1]$ used to split the basic belief assignement of the focal elements on singletons of Ω. It is equal to 1 if $f^I(y) \subseteq A$ and 0 if $A \cap f^I(y) = \varnothing$. Generally, we have $\omega(A,y) + \omega(\complement A,y) = 1$. The opposite operation is quite more complexe. We will present several approaches of this problem according the context. The aim is to associate a belief function to a probability according the type of source which delivers the probability.

B. Subjective source

In the theory of belief functions, the minimal commitment principle is frequently used [12], [16]. The idea is quite simple. To choose a belief function among a set of belief functions and when there is no reason to prefer one to another, we have to choose the least informative one. That assumes there is an ordering to decide which one is the least informative. One ordering commonly used is funded on the communality function. Indeed, we can consider that communality function is a way to measure the non specificity of a belief function and that this function is linked to the conditionning process [28]. It is an interresting *criterion* if we know that *a posteriori* information will be use in the fusion process. Let assume that a source of information delivers a continuous probability density function $Betf$, but that this probability is induced by a bet. That implies this kind of data comes from a subjective point of view. We note $\mathscr{B}Iso(BetP)$, the set of belief functions whose the pignistic transformation gives $BetP$. There is a belief function [10] belonging to $\mathscr{B}Iso(BetP)$ whose the focal elements are the α-cuts of $Betf$ and the credal measure $\mu^{\mathcal{B}(\mathbb{R}^n)}$ is such as:

$$d\mu^{\mathcal{B}(\mathbb{R}^n)}(\alpha) = \lambda\left(f_{cs}^I(\alpha)\right) d\lambda(\alpha) \qquad (16)$$

Theorem III.1. *Among the set of belief functions $\mathscr{B}Iso(BetP)$, the belief function defined by equation (16) is the least committed one for the communality ordering.*

Proof: If $\left(f^I, \mu^{\mathcal{B}(\mathbb{R}^n)}\right) \in \mathscr{B}Iso(BetP)$, we have by construction $pl\left(\complement f^I(\alpha)\right) - \alpha \cdot \lambda\left(f^I(\alpha)\right) \leqslant betP\left(\complement f^I(\alpha)\right)$ (*cf.* figure 1). The belief described by equation (16) reaches the upper bound of this inequality. We deduce that this is the least committed one for the plausibility ordering. As this belief

function is consonant, it is the least committed one for the communality ordering. ∎

We can build the least committed belief function linked to $\mathscr{B}Iso(BetP)$ when the associated probability density function is continuous. For discret frames of discernement or in particular cases of continuous belief functions, this kind of result has already been obtained [28]. When an expert modelizes a phenomenom with a probability density function, we can use this transformation to combine a belief function with a given distribution of probability. Indeed, we assume that the opinion of an expert is quite subjective.
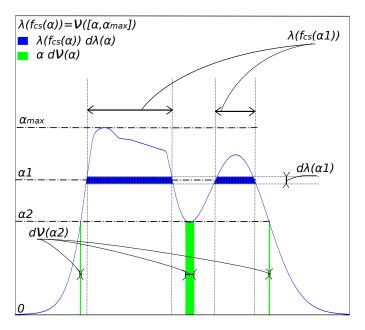

Figure 1. Two different ways to build belief functions from a probability density function $Betf$.

C. Objective source

When we work with an "objective" source of information, we can apply the principle of maximum of necessity. This principle comes from the theory of possibility [19]. The idea is to work with the most informative distribution of possibility (for the necessity ordering) which fulfils the following assumptions. The first one is that the possibility dominate the probability, *i.e.* for all A measurable $\Pi(A) \geqslant P(A)$. The second one is that the ordering must be kept, *i.e.* $P(A) \geqslant P(A') \Leftrightarrow \Pi(A) \geqslant \Pi(A')$. These conditions can be transposed in the framework of belief functions by setting that the plausibility function is equal to the possibility and the necessity is equal to the belief function if we work with a consonant belief function [11]. Finding a belief function which verifies these properties is equivalent to find a nested focal sets family such as for all A belonging to this family, A is the smallest set (for the inclusion ordering) such as $P(A) = \beta$. This sets family corresponds to the confidence sets in theory of probability. If we have as input a continuous probability

density function $Betf$, the focal sets can be described with the α-cuts of this function. We obtain a belief function defined by $\left(f_{cs}^I, \mu^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}\right)$ such as if we adapt the result obtained in [19]:

$$pl^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}(x) = 1 - BetP\left(f_{cs}^I(Betf(x))\right) \qquad (17)$$

i.e.:

$$d\mu^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}(\alpha) = \alpha d\mathcal{V}(\alpha) \qquad (18)$$

with $\mathcal{V}\left([\alpha, \alpha_{\max}]\right) = \lambda\left(f_{cs}^I(\alpha)\right)$ (*cf.* figure 1). When we want to define the mean of a parameter using statistical data, we define a confidence set centered on the empirical mean, $\overline{X_p}$, of the sample of measure. In [2], the authors decide to associate a set of pignistic probabilities to a sample data. Hence, the consonant belief function induced by this sample is the most committed one among those which dominate the set of least commited belief functions linked to each pignistic probabilities. With a such approach, there is a big waste of information. The approach described in [13] seems to be best-adapted. It develops the same idea that we have just described. To define the confidence set, we compute the empiric standart deviation, $\overline{S_p}$. We set that the confidence interval $I_{\beta_{a_t}}$ is an interval such as β_{a_t} is the probability that the true mean is contained by this set. When we work with a small sample set of monodimensional measures (it contains p elements), we have:

$$I_{\beta_{a_t}} = \left[\overline{X_p} - a_t \frac{\overline{S_p}}{\sqrt{p-1}}, \overline{X_p} + a_x \frac{\overline{S_p}}{\sqrt{p-1}}\right] \qquad (19)$$

with $a_t = \left|\overline{X_p} - t\right| \frac{\sqrt{p-1}}{\overline{S_p}}$ and $St(a_t, p-1) = \frac{1+\beta_{a_t}}{2}$ (St refers to the Student's distribution). We can use t belonging to $\left[\overline{X_p}, \infty\right]$ to define an index function. The focal sets are the confidence sets $I_{\beta_{a_t}}$ and:

$$pl^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}(t) = 1 - \beta_{a_t} = 2 \cdot St(-a_t, p-1) \qquad (20)$$

Hence we have:

$$d\mu^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}(t) = \frac{2 \cdot \Gamma\left(\frac{p}{2}\right)}{\Gamma\left(\frac{p-1}{2}\right)\sqrt{\pi(p-1)}}\left[1+\left(\frac{t - \overline{X_p}}{\overline{S_p}}\right)^2\right]^{-\frac{p}{2}} d\lambda(t) \quad (21)$$

with Γ the Euler's function. We can use this approach to model belief function induced by data coming from sensor as it is supposed to be an objectif source of informations.

D. An hybrid source

In either approaches modelling information transmitted by probabilistic information sources, the aim has been to transfer information from singletons to sets. Naturally, two phenomenons have appeared. A first one has been to keep the ordering between the different kinds of representation. If something is more probable than another, it is more plausible. That has lead to use as focal sets the consonant sets' family build with the alpha cut of $Betf$. The second one is that the weight has been splitted on different focal elements according the thrusthworthy of a information source and the size of a focal element. We can use the same approach that the one

describes in [20] and consider that there are several states for the source of information: a subjectif one (R_1) and an objectif one (R_2). Therefore we can consider the frame of discernment $\mathcal{R} = \{R_1, R_2\}$ which sums up the several states of the source. Let be $\mu^{\mathcal{R}}$, the belief function which represents the state of the source of information delivering the pignistic informations. For the sake of simplicity of the notation, we set for all $y \in I$ $\mu^{\mathcal{T}}[X](y) = \mu^{\mathcal{T}}[X](f(y))$. Hence, we have:

$$\begin{cases} \mu^{\mathcal{R}}(R_j) &= \gamma_j \\ \mu^{\mathcal{R}}(\mathcal{R}) &= 1 - \sum_j \gamma_j \end{cases} \qquad (22)$$

Now, using the methods described previouly, we can define the belief function induced by a continuous density of probability function knowing the state of the source. We note these functions $\mu^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}[R_j]$. The belief function linked to the ignorance state is noted $\mu^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}[\mathcal{R}]$ and is equal to $\delta_{\overline{\mathbb{R}}^n}$ (the vacuum belief function) which is a function such as:

$$\begin{aligned} \delta_X: \quad \mathcal{B}\left(\overline{\mathbb{R}}^n\right) &\longrightarrow \{0,1\} \\ B &\longmapsto \begin{cases} 1 & \text{if} \quad B = X \\ 0 & \text{if} \quad B \neq X \end{cases} \end{aligned} \qquad (23)$$

It is used to take into account the reliability of the source [24]. Hence, using the transferable belief model framework, we have [20]:

$$\mu^{\mathcal{B}\left(\mathbb{R}^n\right)}\left[\mu^{\mathcal{R}}, \mu^{\mathcal{B}\left(\mathbb{R}^n\right)}[X \subseteq \mathcal{R}]\right] =$$
$$\left(\underset{X \subseteq \mathcal{R}}{\bigodot} \mu^{\mathcal{B}\left(\mathbb{R}^n\right)}[X]^{\Uparrow\mathcal{B}\left(\mathbb{R}^n\right)\times\mathcal{R}} \bigcirc \mu^{\mathcal{R}\uparrow\mathcal{B}\left(\mathbb{R}^n\right)\times\mathcal{R}}\right)^{\downarrow\mathcal{B}\left(\mathbb{R}^n\right)} \quad (24)$$

with $\Uparrow$, $\uparrow$ and $\downarrow$ which represent the deconditioning, the balooning and the marginalisation process [26]. Hence :

$$\mu^{\mathcal{B}\left(\mathbb{R}^n\right)}\left[\mu^{\mathcal{R}}, \mu^{\mathcal{B}\left(\mathbb{R}^n\right)}[X \subseteq \mathcal{R}]\right] =$$
$$\sum_j \gamma_j \mu^{\mathcal{B}\left(\mathbb{R}^n\right)}[R_j] + \left(1 - \sum_j \gamma_j\right) \mu^{\mathcal{B}\left(\mathbb{R}^n\right)}[\mathcal{R}] \qquad (25)$$

In spite of the fact that we have not dealt with the problem of the evalutation of γ_i, we suppose that an approach similar to the ones found in [14], [20] can be used. In the next section, we will assume that information coming from a sensor is objective and the one coming from an expert opinion is subjective.

IV. DECISION MAKING WITH SENSORS DATA

A. Expression of the problem

To illustrate our approach, we do classification using cinematical data as in [22], [23]. It means we suppose that according the type of target we observe, the distribution of speed is not the same. If we consider m kinds of targets $\mathcal{T} = \{T_1, \cdots, T_m\}$ and an expert defines for each target a distribution of speed (*cf.* figure 2), we can associate to this subjective probability a belief function $\mu^{\mathcal{T}}[T_i]$. In our example, we assume that the sensor delivers an imprecise measure of speed. If the sensor is caracterized by a standart deviation we can assume that the information transmitted by the sensor is a gaussian probability density function. As we consider that the sensor is an objective source of information, we can associate to this source of information a belief function $\mu_{c,x}^{\mathcal{B}\left(\overline{\mathbb{R}}^n\right)}$ with x the measure transmitted by the sensor.
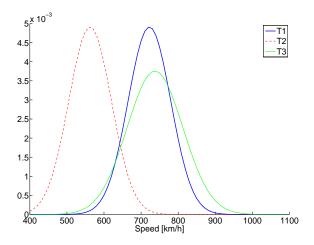
Figure 2. Probabilities induced by the opinion of an expert describing the distribution of speed of several targets.

B. A General Bayes Theorem variation

In [4], the authors decide to use credal inference in order to deal with the lack of information in sensor context. In our study, we prefer to keep the framework of the general bayesian theorem (GTB) [20], [26] because it fulfills the least commitment principle within the transferable belief model. For the sake of simplicity, we set for all y in I that $\mu^{\mathcal{T}}[X](y) = \mu^{\mathcal{T}}[X](f(y))$. If we have the *a posteriori* information that the value of the parameter is in X, we have according the GTB for all A included in $\mathcal{T}$:

$$\mu^{\mathcal{T}}[X](A) = \prod_{T_i \in A} pl^{\mathcal{B}(\overline{\mathbb{R}}^n)}[T_i](X) \prod_{T_i \in \overline{A}} \left(1 - pl^{\mathcal{B}(\overline{\mathbb{R}}^n)}[T_i](X)\right) \quad (26)$$

This equality can be deduce from :

$$\mu^{\mathcal{T}}[X] = \left(\bigodot_{i \in [\![1,m]\!]} \mu^{\mathcal{B}(\overline{\mathbb{R}}^n)}[T_i]^{\Uparrow \mathcal{B}(\overline{\mathbb{R}}^n) \times \mathcal{T}} \bigodot \delta_X^{\mathcal{B}(\overline{\mathbb{R}}^n) \uparrow \mathcal{B}(\overline{\mathbb{R}}^n) \times \mathcal{T}} \right)^{\downarrow \mathcal{T}} \quad (27)$$

with $\delta_X^{\mathcal{B}(\overline{\mathbb{R}}^n)}$ representing the information "the true value of the parameter is in X". In our case, the *a posteriori* information coming from the sensor is modelized by a belief function $\mu_{c,x}^{\mathcal{B}(\overline{\mathbb{R}}^n)} \neq \delta_X^{\mathcal{B}(\overline{\mathbb{R}}^n)}$. Hence, we have:

$$\mu^{\mathcal{T}}\left[\mu_{c,x}^{\mathcal{B}(\overline{\mathbb{R}}^n)}\right] = \left(\bigodot_{i \in [\![1,m]\!]} \mu^{\mathcal{B}(\overline{\mathbb{R}}^n)}[T_i]^{\Uparrow \mathcal{B}(\overline{\mathbb{R}}^n) \times \mathcal{T}} \bigodot \mu_{c,x}^{\mathcal{B}(\overline{\mathbb{R}}^n) \uparrow \mathcal{B}(\overline{\mathbb{R}}^n) \times \mathcal{T}} \right)^{\downarrow \mathcal{T}} \quad (28)$$

We deduce from this equality that:

$$\mu^{\mathcal{T}}\left[\mu_{c,X}^{\mathcal{B}(\overline{\mathbb{R}}^n)}\right](A) = \int_I \mu^{\mathcal{T}}\left[f_{c,X}^{I_c}(y)\right](A) \, d\mu_{c,X}^{\mathcal{B}(\overline{\mathbb{R}}^n)}(y) \quad (29)$$

With this approach, we are able to take a decision in case of model based classification with uncertain data.

C. Classification of target with imprecise sensors' data

Usually, to do classification we use the bayes theorem in a probabilistic framework. If we do not know the probability

a priori to observe a target T_i, we suppose that these events have the same probability. It has an influence on the decision we take when we have a measure (*cf.* figure 3). If we work in
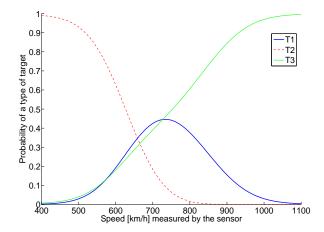

Figure 3. Decision making in classical bayesian framework

the framework of the theory of belief functions, we can use the belief function describes by the equation (29) to define our knowledge by taking into account a measure from a sensor. In order to take a decision on singletons, we can use the pignistic transformation. Hence, we have:

$$BetP(T_i) = \int_{F \cap T_i} \frac{\nu\left(T_i \cap f^I(y)\right)}{\nu\left(f^I(y)\right)} d\mu^{\mathcal{T}}\left[\mu_{c,x}^{\mathcal{B}(\overline{\mathbb{R}}^n)}\right](y) \quad (30)$$

By applying this transformation, we obtain the decision pattern illustrated by the figure 4. As in [23], we see that the lack of in-
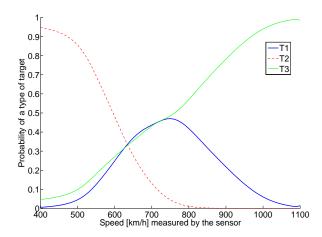

Figure 4. Decision making in belief function framework

formation is best-handled within the theory of belief function. Indeed, as we don't assume an *a priori* knowledge about the occurency of appearance of a target, we take decisions using only the informations we have. Hence, when the situation is ambiguous, we do not favour an option to another.

V. Conclusion

In this paper, we have proposed a way to take into account the imprecision of a source of information. We assume that a source whose the information is model with a distribution of probability could be as well modeled with a belief function. However, to transform a probability into a belief function, we have to take into account the state of the source which provides the information. An objective source is modelized with the belief function the most informative as possible and a subjective one with the least informative belief function linked to the information transmitted by the source. When the source is not reliable, we use the vacuum belief function to modelize the information transmitted by the source. In order to illustrate our work, we have made the assumptions that the opinion of an expert is quite subjective and the information given by a sensor is objective. Hence, using the ballooning, deconditionning and marginalisation process, we have proposed a variation of the General Bayes Theorem in order to take decision in a context of lack of *a prior* information, imprecise *a posteriori* data and model based classification. The results obtain looks like those obtain by B. Ristic [23] but in our case, we can consider in a finess way the information transmitted by the sensor.

Several applications of this work are planned. One of them is to use these results to make classification as in [15]. As we do not use the same procedures to create belief functions from probabilities, we expect to obtain better results. Another one is to use this work to estimate parameters from the data of several sensors. It could be useful to take into account the trustworthy and the reliability of sources of information. Some points need further development. We have to find an efficient method to compute the volume and the probability of a confidence set, to suggest a way to determine the state of a source of information and to propose a rule to keep a consonant belief function after combination.

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