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Theory of belief functions for information combination and update in search and rescue operations

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Abstract – *This paper presents a belief function approach for the location distribution of a search object in an optimal search planning context. We propose several ways to update the negative information obtained following an unsuccessful search mission using a belief functions framework. The discrete search space is defined by cells. We first represent the location information at the cell scale. We then generalize it to the complete grid. We compare different models for updating information on the search object location. We also suggest a way to take into account false alarms to test the expressive power of this framework and deal with a multi-sensor context.*

Keywords: Optimal search, theory of belief functions, false alarm, search and rescue.

1 Introduction

The theory of optimal search was developed during the 2^{nd} world war by B. Koopman [1, 2, 3]. One of the aims of his work was to detect German U-boats in the Atlantic Ocean. Current research efforts apply this theory in a Search And Rescue (SAR) context in order to develop optimal search plans [4, 5]. The goal of an optimal search plan is to make the best use of available search resources in order to locate and rescue people in distress. There are two main components for modeling this problem: *i)* information on the possible location of the missing search object, *ii)* a sensor's capability to detect the search object. This capability depends on the search object type, on the environment where search missions are conducted, and on the sensor and amount of effort spent on searching. In the search and rescue literature, location and detection information are usually modeled using probabilities: the Probability Of Containment (POC) which describes the location distribution of the search object and the Probability Of Detection (POD) which is the conditional probability of detecting a search object with a given amount of effort. A search plan defines the way effort is distributed over a search area. Optimal search planning consists of allocating the available resources in a way to maximize a given performance criterion. Today various sensors are available and are used in the context of searches such as drones for example [6]. Recenlty, some studies [7, 8] have addressed the performance measures of sensors in a SAR context. In this paper, we propose a new way to deal with sensors information based on the theory of belief functions. This theory is based on the work of Dempster and Shafer [9, 10]. It is a mathematical framework useful to represent uncertainty and imprecision. Moreover, it is a powerful tool to merge several sources of information and take into account conflict and ambiguity. In this paper we start by summarizing basic notions of search theory and belief functions theory. We then develop and apply a belief function approach for the classical optimal search problem and extend it to the false alarm cases. Finally, we present simulations results to compare the different models.

2 Optimal search theory

2.1 Modeling of search problem

Let $\mathcal{R}$ be the search area. If $\mathcal{R}$ is continuous, the distribution location of the search object on a search space can be represented by a continuous probability density function f_l defined by [11]:

$$\int_{x\in\mathcal{R}} f_l\left(x\right)dx = \beta \tag{1}$$

where β is a real number between 0 and 1. A β value lower than 1 corresponds to a belief that the search object is outside the search area with a probability of $1-\beta$. If the search area is discrete, we have:

$$\sum_{c\in\mathcal{R}} POC\left(c\right) = \beta \tag{2}$$

where $POC\left(c\right)$, is the probability that the search object is in the cell c.

There are various ways to initialize this location probability distribution [4]. One of them consists in defining a Gaussian function centered on the last known point (LKP). Some methods have been proposed to generate complex probability of containment. These methods use several *scenarii* to define possibility areas [12]. The conditional probability of detecting a search object given that it is in a cell c ($POD\left(c\right)$) depends on several parameters such as the environment, the amount of search effort, the kind of search object and the sensor. To characterize the ability of a sensor to detect a target, we use the lateral range function $\hat{\alpha}\left(r\right)$ [11]. It is the probability that an object, located at a distance r perpendicular to the trajectory of the sensor, will be detected (*cf.* figure 1). When this function is integrated over the distance r , we obtain W, the sweep width:

$$W = 2\int_0^{\infty} \hat{\alpha}\left(r\right) dr \tag{3}$$

In the discrete case, we consider that the sweep width is homogeneous over a given grid cell ($W\left(c\right)$).

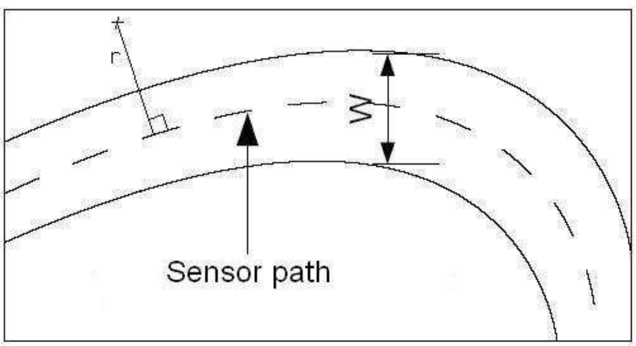

Figure 1: Area swept by the sensor.

A classical lateral range function is $\hat{\alpha}$ [11]:

$$\hat{\alpha}\left(r\right) = \left\{ \begin{array}{lll} 1 & pour & 0 \leq r \leq d \\ 0 & pour & r > d \end{array} \right. \tag{4}$$

with $W = 2d$ (*cf.* figure 1). When a sensor can be described with this law, we call it a definite-range law sensor. There are several ways to measure the search effort [11]. It can defined by a trajectory length, time spent in an area, the cost of a mission, etc. In general, effort is defined as the length of the path followed by the sensor. Let z be this length, V be the speed of the sensor, then $z = V \cdot T$ with T the time spent in an area. The product of z by W gives us an idea of the surface covered by the sensor. Let the sensor follow a definite range law (*cf.* equation (4)), we can use W instead of $\hat{\alpha}\left(r\right)$ to compute the POD defined by the exponential function b as follows:

$$b\left(z\right) = 1 - \exp\left(-zW/A\right) \tag{5}$$

The exponential detection function assumes a random search along the path of the sensor. The detection function given by the equation (5) is a low bound on the probability of detection that can be obtained with different $\hat{\alpha}$. There are many different detection models. For example in [2], Koopman introduces a visual detection model. Following an unsuccessful search mission, the POC is usually updated based on the Bayes' rule. Let n denote the discrete time index, c the cell number, we then have[1]:

$$POC_n\left(c\right) = \frac{POC_{n-1}\left(c\right) \cdot \left(1 - POD_n\left(c\right)\right)}{1 - POS_n} \tag{6}$$

The resulting POC_n is not always necessarily normalized to sum up to 1. In this case, instead of redistributing the POC on all the search area, we assume that the search object is outside the search area. This results in a lower β value. There are many extensions to the basic search problem as defined above such as in [13] where S. Pollock has suggested a way to take into account the false alarm rate of a sensor. Also, some authors make the assumption that the initial value of the sweep width is itself uncertain such as in [14].

2.2 Search planning

To optimally plan a mission, the available effort Ξ must be distributed over the search area in a way that maximizes a performance criterion. Often [4], we try to maximize the probability of find the search object (POS). For one step of planning, we maximize (in this equation, we work on continuous space):

$$\begin{aligned} POS &= \int_{x \in \mathcal{R}} f_l\left(x\right) b\left(\xi\left(x\right)\right) dx \\ \text{with } \Xi &= \int_{x \in \mathcal{R}} \xi\left(x\right) dx \end{aligned} \tag{7}$$

where $\xi\left(x\right)$ is the amount of effort applied on x. Several methods have been proposed to distribute the effort over the search area, depending on the fixed constraints [15]. If we assume that the effort is continuous and infinitely divisible, and that the search object is stationary, de Guenin [16] has proved that for an amount of effort, the POS is maximized if for all x of $\mathcal{R}$:

$$f_l\left(x\right) b'\left(\xi\left(x\right)\right) = \lambda \tag{8}$$

where b' is the derivative in ξ of b and λ is a constant. Therefore, for a fixed λ, by inverting b', we can find the allocation of ξ_λ maximizing the POS on the search area for a global amount of fixed effort. Then we have $\Xi_\lambda = \int_{x \in \mathcal{R}} \xi_\lambda\left(x\right) dx$. The optimization problem is now transformed. Our aim is to find the λ which verifies $\Xi_\lambda = \Xi$. In [17] and [15], the authors present search planning algorithms for moving objects.

[1] *cf.* equation (7) for the definition of POS.

3 Theory of belief functions

A frame of discernment is a finite set of distinct elements noted Ω. The set of subsets of Ω is noted 2^Ω. A basic belief assignment (bba) is a function m^Ω of $2^\Omega \longmapsto [0,1]$ which satisfies $\sum\limits_{A\subseteq\Omega} m^\Omega(A) = 1$. The value of $m^\Omega(A)$ with $A \subseteq \Omega$ represents the basic belief on A. A focal element of m^Ω is a subset A of Ω such as $m^\Omega(A)$ is strictly positive. From a bba m^Ω, we can define following functions:

- belief function:

$$bel^\Omega(X) = \sum_{A\subseteq X, A\neq\emptyset} m^\Omega(A)\,\forall X \in 2^\Omega \quad (9)$$

- plausibility function:

$$pl^\Omega(X) = \sum_{A\subseteq\Omega, A\cap X\neq\emptyset} m^\Omega(A)\,\forall X \in 2^\Omega \quad (10)$$

- pignistic probability [18]:

$$betP^\Omega(X) = \sum_{A\subseteq\Omega, X\in A} \frac{m^\Omega(A)}{|A|\left(1 - m^\Omega(\emptyset)\right)} \forall X \in 2^\Omega \quad (11)$$

Let m_1^Ω et m_2^Ω be 2 bbas. To combine these two bbas, we can use the conjunctive rule of combination [19]. The result of this combination, $m_1^\Omega \otimes m_2^\Omega$ noted $m_{1\otimes 2}^\Omega$, is given by:

$$m_{1\otimes 2}^\Omega(A) = \sum_{X\cap Y=A} m_1^\Omega(X)\, m_2^\Omega(Y), \forall A \subseteq \Omega \quad (12)$$

This rule can be generalized to N bbas m_i^Ω. We then obtain:

$$\underset{i\in[1,N]}{\otimes} m_i^\Omega(A) = \sum_{C_1\cap\ldots\cap C_N=A} \prod_{i\in[1,N]} {m_i}^\Omega(C_i), \forall A \subseteq \Omega \quad (13)$$

Several combination operators have been proposed in the literature, especially to manage conflicting information [20]. Let m^Ω be a bba and h a hypothesis. $m^\Omega[h](A)$ is the value of the bba m^Ω for A if h is true. In this case we have $\sum\limits_{A\subseteq\Omega} m^\Omega[h](A) = 1$. Hence $m^\Omega[h]$ is the bba obtained after conditioning m^Ω on h. Let $\mathcal{T}$ and Ω be two frames of discernment and $m^\mathcal{T}$ be a bba. Let us suppose that we know $m^\mathcal{T}[\omega]$ for all $w \in \Omega$. If $t^* \subseteq \mathcal{T}$ is true, we can deduce from the general Bayesian theorem [21] $m^\Omega[t^*]$[2]:

$$m^\Omega[t^*](A) = \prod_{\omega\in A} pl^\mathcal{T}[\omega](t^*) \cdot \prod_{\omega\in\overline{A}} \left(1 - pl^\mathcal{T}[\omega](t^*)\right) \quad (14)$$

[2]By using equation (10), we have $pl^\mathcal{T}[\omega](t^*) = \sum\limits_{A\subseteq\Omega, A\cap t^*\neq\emptyset} m^\mathcal{T}[\omega](A)$.

4 Representing information related to searching with belief functions

We propose here to use the theory of belief functions to represent the uncertainty on the search object location information. Our objective is to show that this theory is a powerful way to express the available knowledge during a SAR case. We first propose models that do no take into account false alarms. We then generalize the model to include the possibility of false alarms.

4.1 Cell framework

We first represent the available information on a cell's scale. We distinguish two frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, which refers to the presence (p_i) or the absence ($\overline{p_i}$) of the search object in the cell i.
- $D_i = \left\{d_i; \overline{d_i}\right\}$, which refers to the detection (d_i) or the non-detection ($\overline{d_i}$) in the cell i.

We also consider two bbas linked to two sources of information :

- $m_{ci,t}$ which is linked to the information that the sensor in the cell i gives at the instant $t > 0$.
- $m_{i,t-1}$ which is linked to the information at the instant $t-1$ on the presence of the search object in the cell i at the instant t. $m_{i,t-1}$ is a kind of memory of the information on the search object's location and can be updated by combining it with the information given by the sensor at the instant t.

Using the classical model in search theory [11], we can define $m_{ci,t}^{D_i}[p_i]$. Hence we have:

$$\begin{array}{l} {m_{ci,t}}^{D_i}[p_i](d_i) = POD(i) \\ {m_{ci,t}}^{D_i}[p_i]\left(\overline{d_i}\right) = 1 - POD(i) \end{array} \quad (15)$$

This is the belief we have on the occurrence of a detection in cell i given that the object is in that cell.

The bba $m_{i,t-1}$ can be initialized based on the *a priori* information. We use the assumed initial POC to define the bba ${m_{i,0}}^{\Pi_i}$ which is null everywhere except for:

$$\begin{array}{lcl} {m_{i,0}}^{\Pi_i}(p_i) & = & POC(i) \\ {m_{i,0}}^{\Pi_i}(p_i \cup \overline{p_i}) & = & 1 - POC(i) \end{array} \quad (16)$$

As a matter of fact, we only have prior information on the presence in a cell i, hence the rest of the mass can be put on the ignorance $p_i \cup \overline{p_i}$.

At the end of an unsuccessful search mission, we update $m_{i,t-1}$. We must know ${m_{ci,t}}^{\Pi_i}\left[\overline{d_i}\right]$, the belief function on the presence of the search object in the cell i given that there was no detection. We can set:

$$\begin{array}{l} {m_{ci,t}}^{\Pi_i}\left[\overline{d_i}\right](\overline{p_i}) = POD(i) \\ {m_{ci,t}}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i}) = 1 - POD(i) \end{array} \quad (17)$$

Indeed, there was $POD(i)$ chance to detect the search object if it was there. We assume that there is $POD(i)$ chance it is not there after searching. The rest of the mass can be transferred onto the ignorance since the non-detection of the search object does not confirm the absence of the object. Combining $m_{i,t-1}^{\Pi_i}$ with ${m_{ci,t}}^{\Pi_i}\left[\overline{d_i}\right]$, we can update the information on the location of the search object and obtain $m_{i,t}^{\Pi_i}$. By applying the conjunctive rule of combination to merge the information, we obtain:

$$ {m_{i,t}}^{\Pi_i} = {m_{i,t}}^{\Pi_i}\left[\overline{d_i}\right] = {m_{i,t-1}}^{\Pi_i} \otimes {m_{ci,t}}^{\Pi_i}\left[\overline{d_i}\right] \quad (18) $$

The mass assignment to the presence of the search object in a cell decreases each time it is updated following an unsuccessful search of that cell. This behavior is similar to the probabilistic case (*cf.* equation (6)). The mass transferred onto the ignorance (Ω) decreases also when a search effort is applied over a cell. In the theory of probabilities, following an unsuccessful seach, the updated probability of the presence of the search object outside of the search area $(1-\beta)$ increases whereas in the theory of belief functions, the mass on $\emptyset$ increases, because of the conflict between the two sources of information : the *a priori* knowledge about the location of the search object and the result of searches. It is logical to assign a mass to the ignorance since we can never be 100 percent sure that the search object is not in the area searched even following many unsucessful searches. This is due to the fact that there is a nonzero chance of not detecting an object that is located in a cell searched, since detection capability is not perfect.

4.2 Extension to the grid

In this section, we extend the model to the scale of the grid. We consider two frames of discernment:

- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the search object in a cell.
- $D = \left\{d_1; \overline{d_1}; \ldots; d_N; \overline{d_N}\right\}$, the detection or non-detectionof the search object in a cell.

We use the belief function described in model 1. However, instead of using ${m_{i,t}}^{\Pi_i}$, we will use ${m_t}^{\Pi}$. Therefore, the information is available at the scale of grid and not at the scale of cell:

$$ \begin{aligned} {m_0}^{\Pi}(p_i) &= POC(i) \\ {m_0}^{\Pi}\left(\bigcup_{j\in[1;N]} p_j\right) &= 1 - \sum_{j\in[1;N]} POC(j) \end{aligned} \quad (19) $$

We apply the same idea as in the previous section and consider ${m_{ci,t}}^{\Pi}$ instead of ${m_{ci,t}}^{\Pi_i}$. Following an unsuccessful search, the information set is updated based on equation (17). The sensor information if the search is a failure in cell ci can be represented by[3]:

$$ \begin{aligned} {m_{ci,t}}^{\Pi}\left[\overline{d_i}\right]\left(\Pi_{\backslash p_i}\right) &= POD(i) \\ {m_{ci,t}}^{\Pi}\left[\overline{d_i}\right]\left(\Pi_{\cup}\right) &= 1 - POD(i) \end{aligned} \quad (20) $$

We can combine these two bbas to obtain ${m_t}^{\Pi}$, the updated bba of ${m_{t-1}}^{\Pi}$.

We use the conjunctive combination to update the system's information:

$$ {m_t}^{\Pi} = {m_t}^{\Pi}\left[\overline{d_i}\right]_{i\in[1;N]} = {m_{t-1}}^{\Pi} \underset{i\in[1;N]}{\otimes}\left({m_{ci,t}}^{\Pi}\left[\overline{d_i}\right]\right) \quad (21) $$

This representation looks better than the previous one because the information of the sensor and the information on the search object's location are separate. When the information on the search object is updated, a part of the mass is transfered onto the $\emptyset$. This due to the conflict between two sources of information and to the fact that one assumption has not been taken into account: the search object could be outside of the search area. The assignment on $\emptyset$ can be compared to the probability that the search object is outside the search area $1-\beta$. However, this model raises problems : at each update, the number of focal elements of ${m_t}^{\Pi}$ increase.

4.3 Another model on the grid

To prevent having too many focal elements, we propose a new way to represent the information with bbas. Two things are important: the presence of the search object in a cell of the search area and the detection event. We consider three frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, the presence or non-presence of the search object in a cell i.
- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the search object in all the cells.
- $D = \left\{d_1; \overline{d_1}; \ldots; d_N; \overline{d_N}\right\}$, the detection or non-detection in a cell of the grid.

We consider this two bbas:

- $m_{i,t}$, the information given by the sensor in the cell i at the instant $t > 0$ given the information on the system at the instant $t-1$.
- m_t, the information on the system at the instant t.

We initialize m_t:

$$ \begin{aligned} m_0^{\Pi}(p_i) &= POC(i) \\ m_0^{\Pi}\left(\bigcup_{i\in[1;N]} p_i\right) &= 1 - \sum_{i\in[1;N]} POC(i) \end{aligned} \quad (22) $$

[3] We have $\Pi_{\backslash p_i} = \bigcup_{j\in[1;N]-\{i\}} p_j$ et $\Pi_{\cup} = \bigcup_{j\in[1;N]} p_j$.

We then have:

$$\begin{array}{l} m_t^{\Pi_i}(p_i) = m_t^{\Pi}(p_i) \\ m_t^{\Pi_i}(p_i \cup \overline{p_i}) = 1 - m_t^{\Pi}(p_i) \end{array} \quad (23)$$

As in model 1, we get:

$$m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i) = m_{t-1}^{\Pi_i}(p_i) \cdot (1 - POD\,(i)) \quad (24)$$

To reduce the number of focal elements, we put the rest of our belief assignment on the ignorance:

$$m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i}) = 1 - m_{i,t}^{\Pi_i}(p_i) \quad (25)$$

We then apply the result obtained on Π_i to Π:

$$\begin{array}{l} m_{i,t}^{\Pi}\left[\overline{d_i}\right](p_i) = m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i) \\ m_{i,t}^{\Pi}\left[\overline{d_i}\right]\left(\bigcup\limits_{j\in[1;N]} p_j\right) = m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i})\,; \end{array} \quad (26)$$

To update m_t^{Π}, we combine all the $m_{i,t}{}^{\Pi}\left[\overline{d_i}\right]$. To update m_t, we use a conjunctive rule of combination. We then have:

$$m_t^{\Pi} = m_t^{\Pi}\left[\overline{d_i}\right]_{i\in[1;N]} = \otimes_{i\in[1;N]}\left(m_{i,t}{}^{\Pi}\left[\overline{d_i}\right]\right) \quad (27)$$

The bbas have been defined to avoid an explosion of the number of focal elements. The belief stays on the set of pi and the ignorance. When we update the information, a part of the belief is transfered onto the $\emptyset$. This is a consequence of the incomplete frame of discernment and the conflict between the information resulting of an unsuccessful search and the a priori knowledge about the search object's location.

4.4 False alarms

In this section, we propose to take into account the possibility of false alarms in our belief model. Two things are important: the presence of the search object in a cell of the grid and the occurrence of a *bip* by the sensor in the cell. In this model, a *bip* of the sensor is an uncertain detection. We consider three frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, the presence or non-presence of the search object in a cell i.
- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the search object in a cell of the grid.
- $B_i = \left\{b_i; \overline{b_i}\right\}$, a *bip* (b_i) or a no *bip* $(\overline{b_i})$ in the cell i of the grid.

We consider two bbas:

- $m_{ci,t}$, the information of the sensor in the cell i at the instant $t > 0$.
- m_{t-1} the *a priori* information we have on the presence of the search object in cell i at the instant t. We can update it by combining it with the information on the sensor at the instant t. This represents a kind of memory of the search object's location.

We set[4]:

$$\begin{array}{l} m_{ci,t}{}^{B_i}[p_i](b_i) = POD(i) \\ m_{ci,t}{}^{B_i}[p_i]\left(\overline{b_i}\right) = 1 - POD(i) \\ m_{ci,t}{}^{B_i}[p_i]\left(b_i \cup \overline{b_i}\right) = 0 \\ m_{ci,t}{}^{B_i}[\overline{p_i}](b_i) = POD'(i) \\ m_{ci,t}{}^{B_i}[\overline{p_i}]\left(\overline{b_i}\right) = 1 - POD'(i) \\ m_{ci,t}{}^{B_i}[\overline{p_i}]\left(b_i \cup \overline{b_i}\right) = 0 \end{array} \quad (28)$$

According to the general Bayesian theorem (14), we have:

$$\begin{array}{l} m_{ci,t}{}^{\Pi_i}[b_i](p_i) = POD(i)\,(1 - POD'(i)) \\ m_{ci,t}{}^{\Pi_i}[b_i](\overline{p_i}) = POD'(i)\,(1 - POD(i)) \\ m_{ci,t}{}^{\Pi_i}\left[\overline{b_i}\right](p_i) = POD'(i)\,(1 - POD(i)) \\ m_{ci,t}{}^{\Pi_i}\left[\overline{b_i}\right](\overline{p_i}) = POD(i)\,(1 - POD'(i)) \\ m_{ci,t}{}^{\Pi_i}[b_i](\Pi_i) = 1 - \left(m_{ci,t}{}^{\Pi_i}[b_i](p_i) + m_{ci,t}{}^{\Pi_i}[b_i](\overline{p_i})\right) \\ m_{ci,t}{}^{\Pi_i}\left[\overline{b_i}\right](\Pi_i) = 1 - \left(m_{ci,t}{}^{\Pi_i}\left[\overline{b_i}\right](p_i) + m_{ci,t}{}^{\Pi_i}\left[\overline{b_i}\right](\overline{p_i})\right) \end{array} \quad (29)$$

If the probability of having a false detection is equal to zero, we will obtain the same results given by the model presented in section 4.2. Taking into account the false alarms justifies the representation of ignorance. Indeed, with a *bip*, we can have the presence but also the absence of the search object. For example, consider the situation where several drones are searching for the same search object, we can combine the information obtained simultaneously on several cells [6]. We apply the following ballooning:

$$\begin{array}{l} m_{ci,t}^{\Pi}[\beta_i](p_i) = m_{ci,t}^{\Pi_i}[\beta_i](p_i) \\ m_{ci,t}^{\Pi}[\beta_i]\left(\bigcup\limits_{j\in[1;N]-\{i\}} p_j\right) = m_{ci,t}^{\Pi_i}[\beta_i](\overline{p_i}) \\ m_{ci,t}^{\Pi}[\beta_i]\left(\bigcup\limits_{j\in[1;N]} p_j\right) = m_{ci,t}^{\Pi_i}[\beta_i](p_i \cup \overline{p_i}) \end{array} \quad (30)$$

with $\beta_i \in B_i$. We initialize m_t^{Π} with the POC:

$$\begin{array}{lcl} m_0{}^{\Pi}(p_i) & = & POC(i) \\ m_0{}^{\Pi}\left(\bigcup\limits_{i\in[1;N]} p_i\right) & = & 1 - \sum\limits_{i\in[1;N]} POC\,(i) \end{array} \quad (31)$$

Therefore, we can combine these bbas to update $m_{t-1}{}^{\Pi}$.

We use the conjunctive rule of combination to update the information:

$$m_t{}^{\Pi} = m_t{}^{\Pi}[\beta_i]_{i\in[1;N]} = m_{t-1}{}^{\Pi} \underset{i\in[1;N]}{\otimes} m_{cj,t}{}^{\Pi}[\beta_j] \quad (32)$$

[4]ex: $m_{ci,t}{}^{B_i}[p_i](b_i)$ is the value of the bba for b_i given p_i at the instant t, on the frame of discernment B_i.

with $\beta_i \in B_i$. With the theory of belief functions, we can easily combine data from several sources of information. However, the size of the power set with this kind of representation is an important problem because the size of the problem increases very quickly.

5 Experimentation and results

In order to evaluate how the different uncertainty models impact optimal search planning results, we present some simulations results.

5.1 Comparison of the different models without false alarms

The theory of belief functions is a powerful tool to represent uncertainty. A way to compare it with the probability theory is to use the pignistic probability (*cf.* equation (11)). Hence, instead of maximizing the POS like in the probabilistic case, we will allocate available effort in a way to maximize the average product of the pignistic probability of the event p_i by the pignistic probability of d_i given p_i. Therefore, in model 1 we will allocate the effort in order to maximize [5]:

$$\sum_{i\in[1;N]} betP^{\Pi_i}_{i,t-1}(p_i)\, betP^{D_i}_{ci,t}[p_i](d_i)$$
$$= \sum_{i\in[1;N]}\left(\frac{m^{\Pi_i}_{i,t-1}(p_i)}{1-m^{\Pi_i}_{i,t-1}(\emptyset)} + \frac{m^{\Pi_i}_{i,t-1}(p_i\cup\overline{p_i})}{2\left(1-m^{\Pi_i}_{i,t-1}(\emptyset)\right)}\right)POD(i) \tag{33}$$

For the models 2 and 3 we will maximize:

$$\sum_{i\in[1;N]} betP^{\Pi}_{t-1}(p_i)\, betP^{D_i}_{ci,t}[p_i](d_i)$$
$$= \sum_{i\in[1;N]}\left(\sum_{A\in 2^\Pi, p_i\in A}\frac{m^{\Pi}_{t-1}(A)}{|A|\left(1-m^{\Pi}_{t-1}(\emptyset)\right)}POD(i)\right) \tag{34}$$

We assume that the sensor detection model follows the exponential detection function (*cf.* equation (5)). To initialize the search, we must assume a grid of POC, of W, a cell area and an available effort Z .

If the effort is assumed to be continuous, we can use the de Guenin's algorithm to allocate Z over the grid [15]. In order to compare the models, we observe how the effort distribution varies over the grid. We can use the standard deviation to verify if the effort distribution is spatially homogeneous. In order to decide in which model the information on the search object is degraded the fastest, we only use the information on the location of the search object to allocate effort. We therefore fix W at the same value in all the cells of the grid.

We used a grid of dimension 4×4 for the test, where all the cells have an area equal to 1. We fixed W equal to 1 over the grid. We assumed that the POC is defined using a bivariate Gaussian distribution over the grid.

[5] *cf.* model 1 for notations.

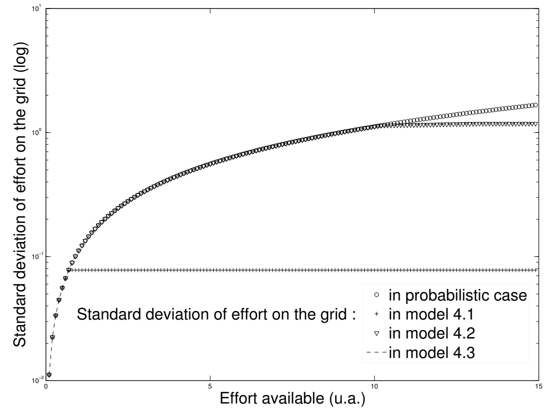

Figure 2: Standard deviation of the effort on the grid according the global amount of effort for several kinds of model.

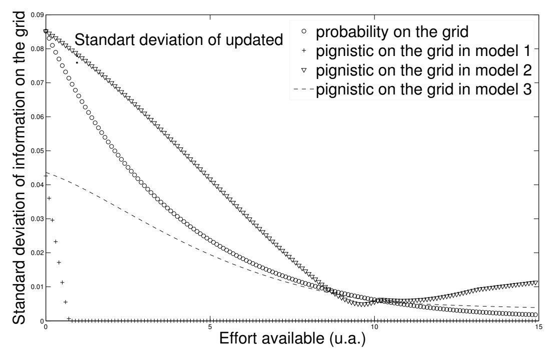

Figure 3: Standard deviation of the information of the presence of the plane on the grid according the global amount of effort for several kind of modelization.

We see on figure 2 that with models 2 and 3, the standard deviation of effort distribution is the same (*cf.* figure 2). This is because we use in these models the same information to distribute the effort. In model 1, the standard deviation is less important, so the effort is more homogeneously distributed over the grid. For each model, there is an effort threshold above which adding more effort will not change the standard deviation of the effort distribution (*cf.* figure 2). This implies that the information we use to distribute the effort is not relevant anymore. As a matter of fact, this happens when the spatial standard deviation of the location information on the search object following an update is near zero (*cf.* figure 3). In model 2, after this threshold, the curve increases. This is due to the fact that the basic belief assignement on each subset of Π becomes so small that the computer's precision is not accurate enough.

5.2 Model with false alarms

We study the consequences of a *bip* or a no *bip* in this model. We begin by defining a grid of POC (*a priori*), of POD and of POD'. We chose to use a grid of dimension 4×4 in which the pignistic probability is uniform. We fixed two ROC curves which represent the behavior of a sensor used by day or by night (*cf.* figure 4). We

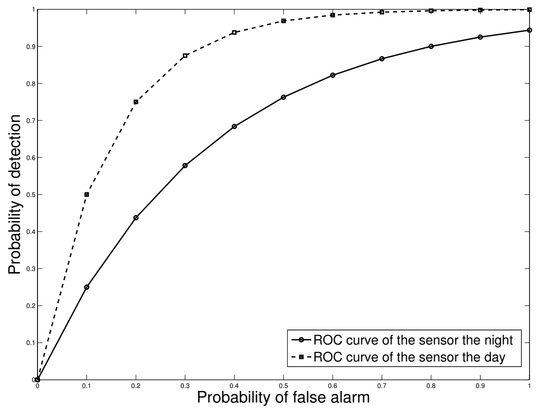

Figure 4: ROC curves of the sensor.

assume that the sensor scans the search area for each point of the ROC curves and we then generate random *bips* for 50 different experiments. Thereby, we obtain the mean number of *bips* for each point of the curve (*cf.* figure 5) and the spatial distribution of pignistic probability at the end of search (*cf.* figure 6). We notice that the most important element is not the number of bips but the difference between POD et POD', because the bigger the difference, the bigger is our certainty about the meaning of a bip.

6 Conclusions and perspectives

In this paper, we have proposed a new approach, based on the belief functions framework, to deal with the location uncertainty within the optimal search problem. With the belief function theory, we have a powerful way to merge information from several sources on the location of the search object. Further work is necessary to improve our model. For example, a first step could be to use the least committed [22] isopignistic basic belief assignments associated to the POC and the POD. Moreover, we could also apply the continuous belief function theory [23]. As a matter of fact, since the complexity in the discrete case is exponential, using a continuous approach can make the computations easier because we can avoid the problem associated with the large number of focal elements.

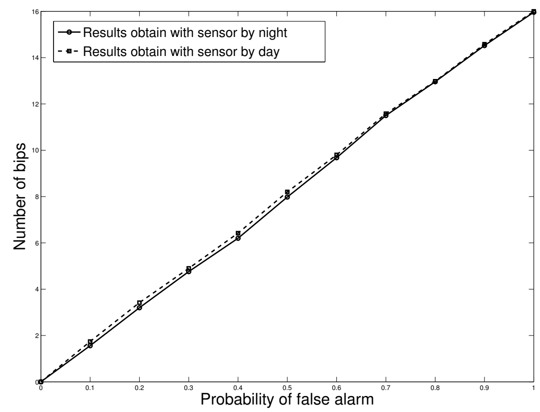

Figure 5: Number of *bips* according to the configuration of the sensor.

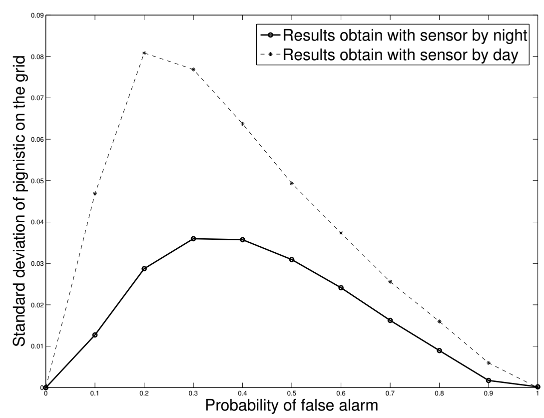

Figure 6: Standard deviation in the spatial repartition of pignistic probability according the configuration of the sensor.

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